MULTIDISCIPLINARY OPTIMIZATION METHODS FOR PRELIMINARY DESIGN

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SUMMARY

An overview of multidisciplinary optimization (MDO) methodology and two applications of this methodology to the preliminary design phase are presented. These applications are being undertaken to improve, develop, validate and demonstrate MDO methods. Each is presented to illustrate different aspects of this methodology. The first application is an MDO preliminary design problem for defining the geometry and structure of an aerospike nozzle of a linear aerospike rocket engine. The second application demonstrates the use of the Framework for Interdisciplinary Design Optimization (FIDO), which is a computational environment system, by solving a preliminary design problem for a High-Speed Civil Transport (HSCT). The two sample problems illustrate the advantages to performing preliminary design with an MDO process.

1.0 INTRODUCTION

Multidisciplinary optimization (MDO) methods and preliminary design are terms for processes that can have different interpretations for both engineers and designers, depending on the background and area of expertise of the individual. This paper addresses preliminary design from an aerospace point of view. The paper is organized as follows. First, definitions are given for the different levels of aerospace design that are discussed in this paper. A discussion of MDO and its conceptual elements follows, and then MDO requirements for preliminary design are presented. Finally, two applications of MDO methodology are illustrated for use in the preliminary design stage.

In this report, the aerospace design process is broken down into three major levels: conceptual design, preliminary design, and detailed design. The detailed design level involves designing for manufacturing and assembly and is beyond the scope of this paper. A short description of the conceptual and preliminary design level follows.

Conceptual design involves the exploration of alternate concepts for satisfying vehicle design requirements. Trade studies between vehicle designs are made with system synthesis tools, which encompass a broad range of disciplines (Fig. 1). Typical system synthesis tools contain extremely simple vehicle geometry descriptions and have shallow and uneven levels of analyses within the disciplines. Typical figures of merit for evaluating the relative importance of design parameters on the conceptual vehicle design are system performance and system cost.

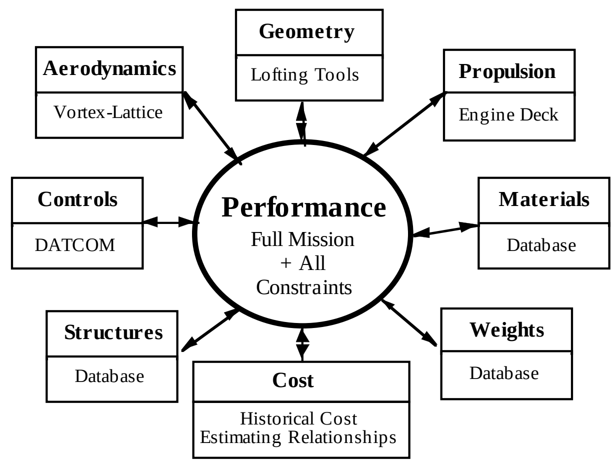

Fig. 1. Sample conceptual design disciplines and tools.

Numerous examples of conceptual design systems or methods can be found that have been used by companies and government organizations. Many companies have their own conceptual design system and processes, which contain proprietary data for the predicted cost and performance of a product. An example of a conceptual design method for aerospace vehicles is the Flight Optimization System (FLOPS).[1] For the conceptual design of hypersonic vehicles with airbreathing engines, two examples of methods under development are Holist[2,3] and PrADO-Hy.[4]

After a vehicle design concept is selected, the design and analysis process evolves from the vehicle concept toward the actual components and subsystems of the vehicle. Specialists become involved in the design and analysis of the different subsystems. Each specialist uses increased detail in their discipline, which results in a more limited interaction with other disciplines. The geometry is described in enough detail to define the subsystem but not enough to specify each assembly. Sophisticated discipline analyses, along with design by analysis or limited optimization, are often used. Typical figures of merit for preliminary design are subsystem performance, size, weight, and cost (Fig. 2).

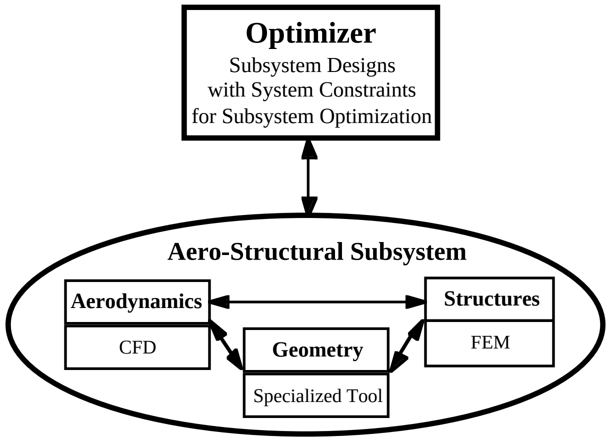

Fig. 2. Preliminary design using an optimization process.

2.0 MDO CONCEPTUAL ELEMENTS

The MDO methodology coherently exploits the synergism of mutually interacting phenomena to improve the designs of complex engineering systems. This process can be used at any stage of a design (i.e., conceptual, preliminary, or detailed design). Typical objectives consist of one or more of the following: improving performance, lowering cost, or shortening the development time for products. The current state of the art in MDO was reviewed in an AIAA white paper[5] in 1991, 1995 ICASE and NASA Langley Workshop,[6] and by Sobieszczanski-Sobieski and Haftka.[7] Additional information containing recent developments can be found in the 6th symposium on Multidisciplinary Analysis and Optimization sponsored by AIAA, NASA and ISSMO.[8]

The MDO methodology is much broader than multidisciplinary analysis and involves various design-improvement strategies, including formal optimization. Notionally,

$$\Delta_{Design} = \left(\sum_i \Delta_{Discipline\ i}\right) + \Delta_{MDO} \qquad (1)$$

where the sum on the right-hand side refers to contribution from individual disciplines and the “Δ_{MDO} ” includes the contributions from the integration of the disciplines. Table 1, which is based on the discussion by Sobieszczanski-Sobieski,[9] is a “taxonomy” of the MDO discipline. A brief description of each element in the table is given, column by column.

Table 1. MDO Conceptual Elements

Information science and technology	Design-oriented multidisciplinary analysis	MDO
• Product data models • Data and software standards • Data management storage and visualization • Software engineering practices • Human interface	• Mathematical modeling • Cost versus accuracy trade-off • Smart reanalysis • Approximations • Sensitivity analysis	• Discipline optimization • Decomposition • Design space search • Optimization procedures

2.1 Information Science and Technology

The general category of information science and technology refers to the information infrastructure that enables MDO; many of the new developments have originated in computer science technology advancements. A prerequisite for even multidisciplinary analysis is the availability of common **Product Data Models**, which are the basis for the geometry and discretization models that are consistent across disciplines. Application of optimization requires that the basic model description be parametric. An elementary example of a parametric surface geometry model that is implemented in a commercial computer-aided design (CAD) system is shown in Fig. 3.[10] This particular model is for a High-Speed Civil Transport (HSCT) and is representative of a capability that is being developed to integrate a parametric CAD system into the preliminary design example covered later in section 4.2.

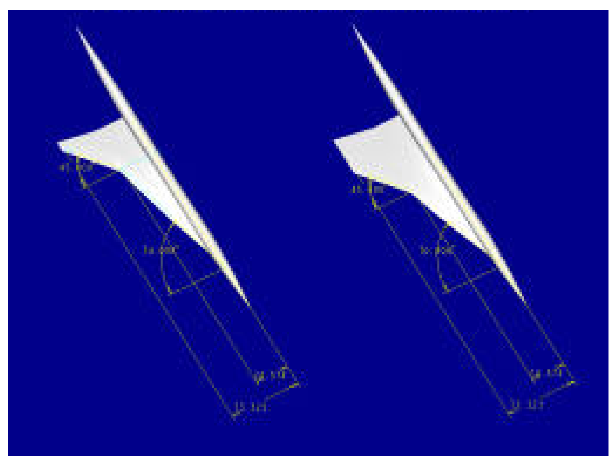

Fig. 3. Sample of CAD parametric geometry model.

Data and Software Standards are necessary for software creation, validation, and documentation, as well as for the definition and archival of data in order to facilitate the use, reuse, and efficient integration of the product software in multidisciplinary systems**. Data Management Storage and Visualization** refers to tools for collecting, storing, managing, visualizing, and documenting data generated in a multidisciplinary analysis or design process. **Software Engineering Practices** are methods for producing, operating, maintaining, and documenting robust software for multidisciplinary applications. The **Human Interface** element is perhaps the most challenging element. Tools are needed to facilitate integration of disciplinary software in multidisciplinary processes; to direct, redirect, and monitor process execution; and, in general, to maximize the potential for human direction of the process.

2.2 Design-Oriented Multidisciplinary Analysis

The adjective *design-oriented* refers to those additional features that must be present in analysis tools if they are to be truly useful in supporting the design process and not merely in producing isolated analyses. The salient issues involve both capability and efficiency, and the MDO developments comprise basic mathematical and algorithmic advances in analysis capability. **Mathematical Modeling** may be required to enable the incorporation of new disciplines into the MDO setting; these models must be able to predict system disciplinary response and measure the impact of changes in other disciplines on disciplinary response. **Cost Versus Accuracy Trade-Off** methods enable trade-offs to be made between computational cost and computational accuracy as necessary. **Smart Reanalysis** refers to efficient reanalysis techniques that minimize the computations required in simulating a system with perturbed input parameters. **Approximations** are generic tools for reliably approximating system disciplinary or multidisciplinary response by using zero- and, potentially, higher order system information.[11] **Sensitivity Analysis** must be supplied via efficient numerical tools that predict the effect that changes in input parameters have on disciplinary and system responses.[12] Recent work on the application of automatic differentiation technology to Navier-Stokes codes to extract efficient gradients (sensitivities) with respect to design variables is compared to the conventional finite difference approach in Fig. 4.[13]

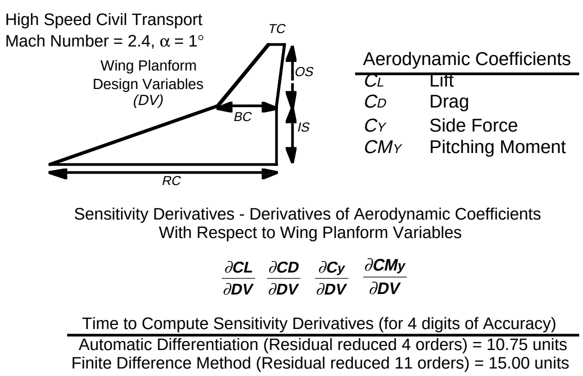

Fig. 4. Sensitivity analysis with automatic differentiation of Navier-Stokes flow code.

2.3 MDO

The distinguishing characteristic of the MDO tools in Table 1 is the use of formal optimization methods to achieve design improvement. **Discipline Optimization** is related to MDO insofar as it uses pathfinding developments of optimization in selected disciplines that eventually target the multidisciplinary application. **Decomposition** methods examine the decomposition of complex processes to identify the best sequence of subprocesses for numerical and computational efficiency and to track the effect that changes in the input to one subprocess have on the output of other subprocesses. One multilevel optimization method that exploits disciplinary optimization techniques in a multidisciplinary setting is illustrated in Fig. 5. Braun[14] has demonstrated collaborative optimization for launch vehicle design.

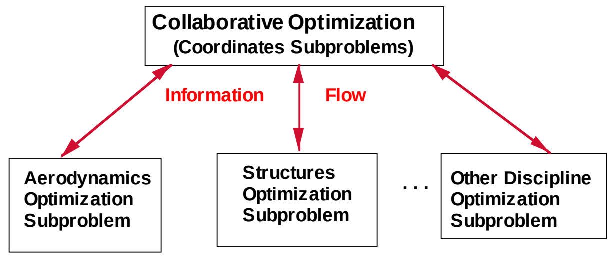

Fig. 5. Decomposition with the collaborative optimization approach.

Design Space Search algorithms facilitate exploration of large design spaces, including those that may be characterized by discrete variables, discontinuous functions, or disjoint subspaces. **Optimization Procedures** in this context refer to optimization algorithms in multidisciplinary procedures that efficiently generate improved designs for multidisciplinary systems. Typical procedures may combine search algorithms, decomposition methods, and approximations. For example, the direct method approach to optimization combines expensive analyses and sensitivity analyses for every step of the optimization algorithm. This contrasts with the indirect method, in which the optimization method is instead coupled with a local approximation which can be based on zero- and first-order information (Fig. 6). The latter approach is far more common in current MDO applications. The indirect method typically requires more optimization cycles to converge than the direct method but is often more efficient because each optimization cycle is less expensive.

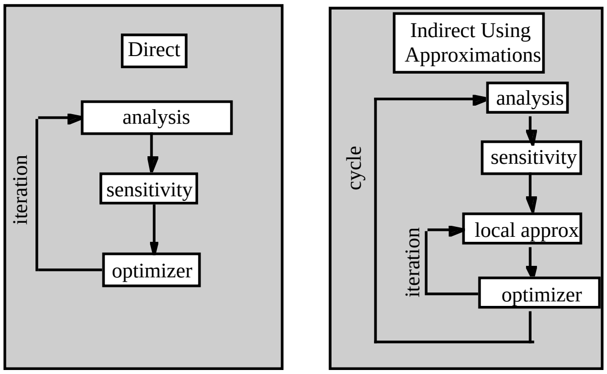

Fig. 6. Comparison of direct and indirect optimization procedures.

3.0 MDO REQUIREMENTS FOR PRELIMINARY DESIGN

The introduction of MDO at the preliminary design stage requires an appropriate information infrastructure, design-oriented multidisciplinary analysis tools, and efficient, robust MDO strategies.

The information infrastructure should provide adequate computer horsepower for the increased computational demands; a common geometry model; a collaborative work environment; an effective means of integrating analysis tools into the overall framework; management of distributed computing; the ability to handle proprietary codes and legacy codes; effective tools for presenting the results; and configuration control.

At the preliminary design level, physics-based methods have generally replaced historical databases in discipline analyses. Typically, a physics-based discipline analysis requires a suite of tools, including pre-processors (e.g., geometry modeller, grid generator, translators for input data needed from other disciplines), an analysis code, and postprocessors (e.g., visualization, translators for output data needed by other disciplines). In an MDO environment, the typical discipline output needs to be supplemented with sensitivity information, and this process needs to be automated. Well-posed interfaces are required between disciplines. This requirement may necessitate more than straightforward interpolation between, for example, aerodynamic surface pressures and structural loads and, in the reverse direction, between structural displacements and surface geometry. The transformations must respect physical principles, and the entire cycle must be consistent.

The basis for any MDO strategy is the problem definition. You need a strategy because the multidisciplinary analysis is usually expensive and difficult to develop. A clear statement of the design variables (and their allowable ranges), the objective function(s), and the constraints is necessary. The appropriate MDO strategy depends on such factors as the mix of continuous and discrete variables; the strength of the interdisciplinary interactions; the separability of the constraints with respect to the design variables; the susceptibility of the analysis tools to algorithmic noise; computational requirements of the analysis; and last, but certainly not least, the compatibility of the MDO strategy with the existing organizational structure and culture.

4.0 PRELIMINARY DESIGN EXAMPLES USING MDO

Two different applications have been selected to illustrate preliminary design with MDO. The first example uses a direct optimization procedure for the multidisciplinary design of an aerospike rocket nozzle.[15] The second example is an HSCT design that utilizes an indirect optimization approach.[16]

4.1 MDO Applied to Aerospike Rocket Nozzle

A multidisciplinary analysis of an aerospike nozzle has been developed to evaluate MDO strategies and new preliminary design processes. This effort was part of a formal collaboration between NASA Langley Research Center and the Rocketdyne Division of Boeing North American, Inc. The linear aerospike rocket engine is the propulsion system proposed for the X33 and the VentureStar[17] (Fig. 7) reusable launch vehicles (RLV).

Fig. 7. VentureStar RLV with linear aerospike propulsion system.

The aerospike rocket engine consists of a rocket thruster, cowl, aerospike nozzle, and plug base region (Fig. 8). The aerospike nozzle is a truncated spike or plug nozzle that adjusts to the ambient pressure[18] and integrates well with launch vehicles. The flow-field structure changes dramatically from low to high altitude on the spike surface and in the base-flow region.[19-20] Additional flow is injected in the base region to create an aerodynamic spike[21] (thus, the name "aerospike"), which increases the base pressure and the contribution of the base region to the aerospike thrust.

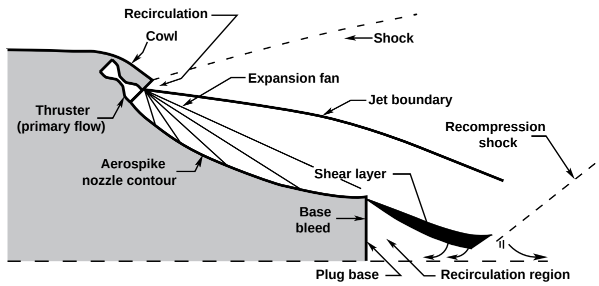

Fig. 8. Aerospike components and flow-field characteristics.

Thrust and nozzle wall pressure calculations were made by using computational fluid dynamics (CFD) and were linked to a structural finite-element analysis to determine nozzle weight and structural integrity (Fig. 9). A mission-averaged specific impulse (ISP) and engine thrust-to-weight ratio were calculated and used to determine vehicle gross-liftoff-weight (GLOW) from data that were defined in the conceptual design stage. The computational time for computing the thrust using CFD techniques was approximately 20 sec, and the computational time for computing the weight using finite element method (FEM) was approximately 35 sec on a Sun UltraSPARC. Approximately 4 months were required to develop and integrate the discipline codes to obtain a multidisciplinary analysis. A significant part of this development was devoted to developing suitable procedures for automating the use of the commercial FE code.

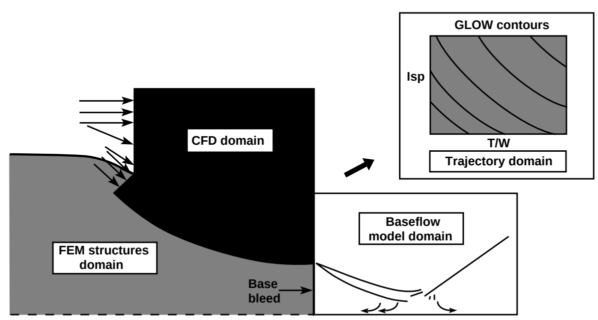

Fig. 9. Multidisciplinary domain decomposition.

The multidisciplinary analysis was integrated with an optimization code that allowed investigation of the multidisciplinary feasible[22] (MDF) strategy. Two different methods of design were investigated. The first method involved the development of a preliminary design by optimizing the disciplines separately. The optimal thrust and nozzle weight were then used to calculate the GLOW. This first method is a model of a typical design approach. The second method utilized the MDF formulation and minimized the GLOW directly, subject to satisfying the structural constraints. The MDF method was applied to a case in which the nozzle length was held constant and one in which this length was varied. The gradient-based optimization method, CONMIN,[23] was used in all cases. A typical optimization problem was solved in 1 to 3 days on a typical workstation and required approximately 300 to 600 multidisciplinary analyses.

The design parameters included 5 geometry variables (Fig. 10) and 14 structural variables (Fig. 11). The initial geometry design variables were selected from previous design studies on aerospike nozzles that used conventional design methods and were expected to be close to an optimized aerodynamic shape.

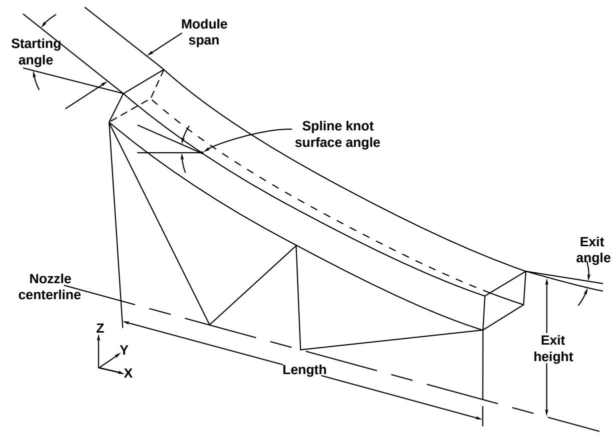

Fig. 10. Aerospike nozzle geometry design parameters.

The number of structural design variables was reduced by mapping some of the design variables with common attributes into a single design variable. In particular, the thickness of the I-beams was made to be the same in each structural box, and the six structural supports were required to have the same radius and wall thickness. The initial values selected for the structural design parameters resulted in a structural design that was infeasible (some constraints were violated). The structural design concept for the aerospike nozzle was generated explicitly for this study and does not relate to a structural configuration that has previously been designed or studied.

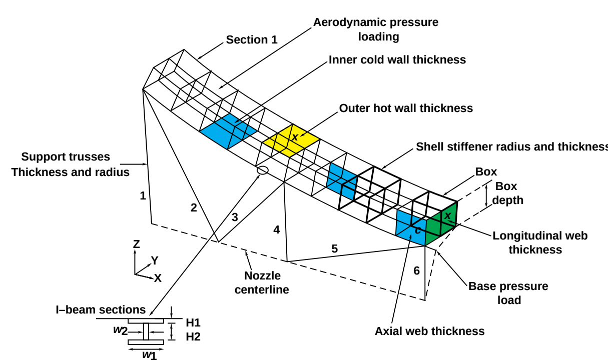

Fig. 11. Aerospike nozzle structural design parameters.

A multidisciplinary design was obtained each discipline separately for a fixed nozzle length (Fig. 12). The MDO design resulted in an improvement of approximately 5 percent in the GLOW over that of the single-discipline optimized solution. The improvement was obtained by reducing the nozzle thrust, which resulted in a lower pressure loading on the nozzle structure and a lower nozzle weight.

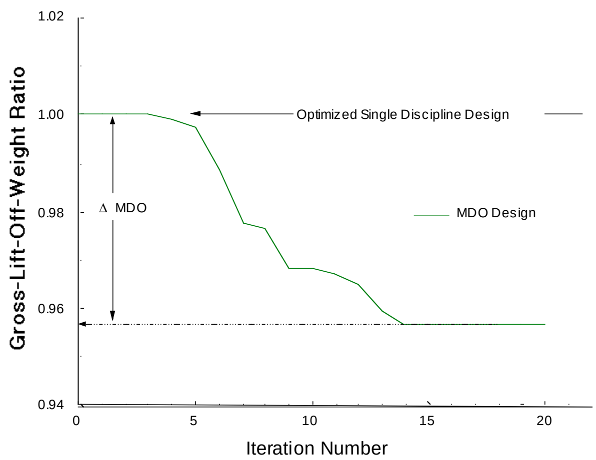

Fig. 12. Improvement of GLOW Using MDO.

The advantages of the MDO approach were evident both in the improvement that was realized in the design objective and the ease with which the multidisciplinary design variables, such as nozzle length, were included in the design process.

4.2 HSCT Preliminary Design with FIDO

The Framework for Interdisciplinary Optimization (FIDO) is being developed to demonstrate multidisciplinary computations on a networked, heterogeneous cluster of workstations, vector computers, and massively parallel computers. This project is one of NASA's contributions to the national High Performance Computation and Communication Program (HPCCP).[24] The FIDO system has been applied to a simplified case of an HSCT design (Fig. 13).

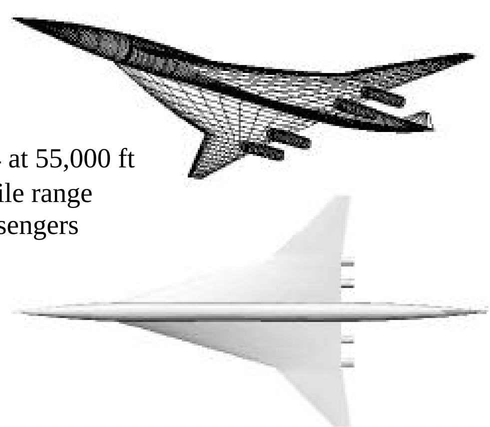

Fig. 13. Preliminary design geometry for HSCT.

The concept that is being used for FIDO is coarse-grained parallelism, with instances in which disciplinary codes are run on separate processors (including, loosely fine-grained parallel computers) under the control of an executive on another processor and with automatic data exchange through a centralized data manager (Fig. 14).

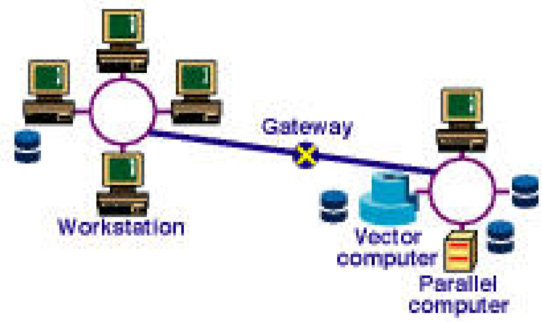

Fig. 14. Heterogeneous distributed computing environment.

The conceptual environment in which the distributed discipline and system-service codes are run is illustrated in Fig. 15. Each of these codes may be run on a separate workstation or on a high-performance computer; the communications and synchronization is handled through the FIDO communication library, which is based on a PVM (parallel virtual machine) utilities. The triangular, rectangular, and hexagonal modules at the top of Fig. 15 are system-service modules that do not change as the design problem changes; the rounded rectangular modules at the bottom of the figure represent the problem-specific computational disciplines and the application-specific, user developed Master module that controls the sequencing for a particular problem. The user interacts with the system through the graphic user interface (GUI), which displays the state of the FIDO system at all times from start-up to completion of a run. The GUI also provides access to multiple system capabilities: the setup module is used to select the design problem,

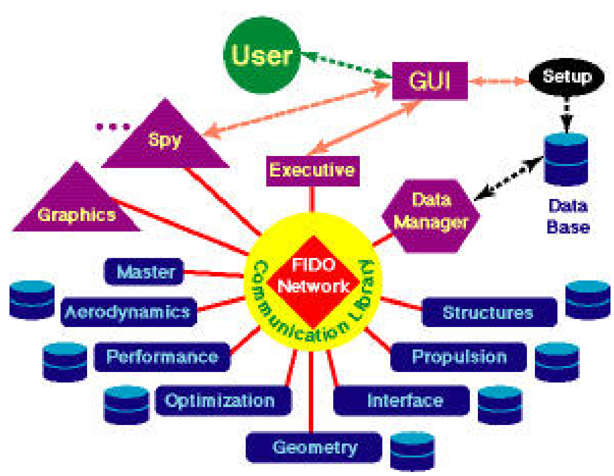

Fig. 15. Executive system of FIDO.

initial configuration, and design conditions and controls; the executive module starts a communication server on each of the distributed computers, distributes initial data and executables, and starts execution of a problem; and the spy module allows the designer, as well as multiple remote users, to query the centralized database and invoke external graphics programs in order to plot selected output data. The data manager provides a centralized access service for the storage and retrieval of global data during a run of the FIDO system.

To update the grids for the aerodynamic and structural FEM analysis, the baseline geometry and the design variables updated in the design process are used to modify the baseline geometry descriptions (Fig. 16). The initial aero analysis provides the drag polars, and the initial structural analysis provides the structural weights. Based on the wing shape specified at the start of cruise condition, the unloaded shape (also called the jig shape) is determined through a structural analysis with the aerodynamic and inertial loads removed. An iterative aeroelastic analysis is used to determine the shape of the configuration at the end of cruise, based on the fuel weight determined through a performance analysis. This performance analysis uses the drag polars and engine fuel-consumption characteristics at the start and end of cruise. The structural deflections, stresses, and weights are provided as input to the optimizer, along with design sensitivity derivatives that are determined during the analyses. The optimizer determines updated values for the design variables, with the objective of minimizing take-off-gross weight subject to a set of design constraints. After the designer reviews and accepts or modifies the design variables, the process begins again if convergence to a minimum weight has not been achieved.

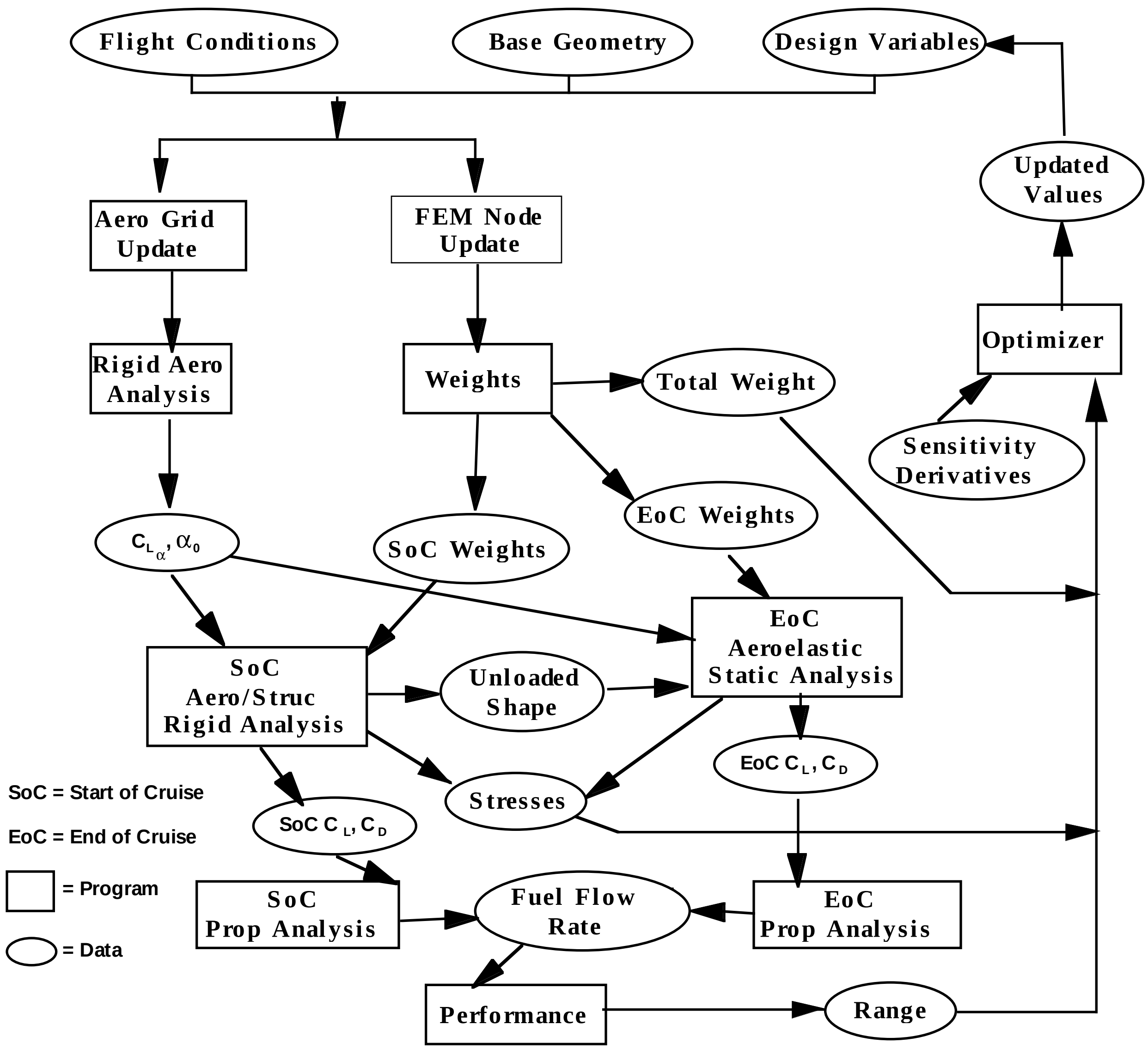

Fig. 16. Problem diagram for HSCT.

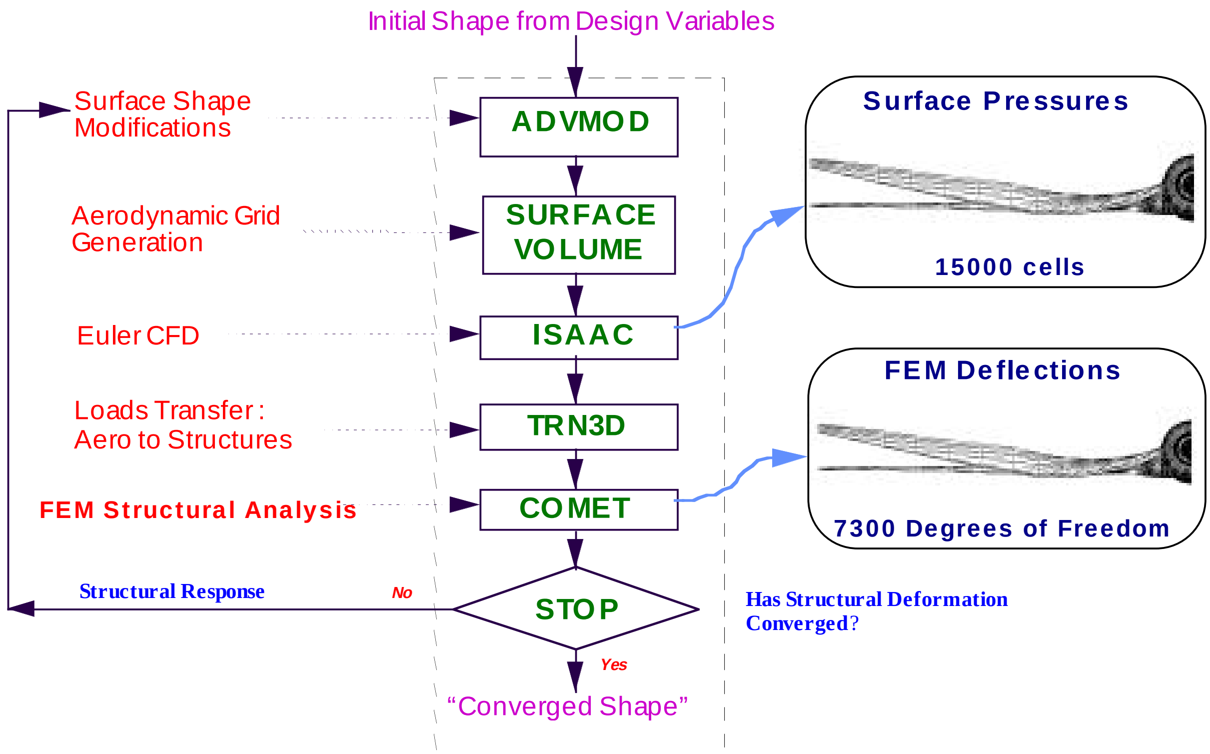

Fig. 17. Key steps in FIDO aeroelastic loop.

An aeroelastic loop implements the tight coupling between the computationally intensive aerodynamic and structural analysis programs (Fig. 17). At the beginning of the loop, the program ADVMOD uses the recently updated design variables to modify the "rubberized" aircraft surface grid and the structural FEM grid in a consistent manner. (The topologies and connectivities of the grids are maintained.) A special-purpose program is then invoked to generate a CFD volume grid that is suitable for the marching Euler program ISAAC[25]. After the aerodynamic calculations are computed, the program TRN3D is used to accurately transfer the surface pressures into FEM node forces for use in the structural analysis code COMET[26]. After COMET determines the nodal deflections that correspond to the aerodynamic and inertial forces, the movement of selected nodes is used to update the deformation of the aircraft surface grid, and the loop is ready to begin again if the shape has not converged.

The weight history from a run of the FIDO system in which the aircraft skin-thickness distribution is represented by two polynomial equations (one each for the wing both inboard and outboard of the break in the leading-edge sweep) is shown in Fig. 18. The coefficients in the polynomials are the design variables for this run. The objective is the minimization of total weight subject to constraints on the material stresses and structural deflections. The plot shows the smooth and converging reduction of the total weight as the design progresses.

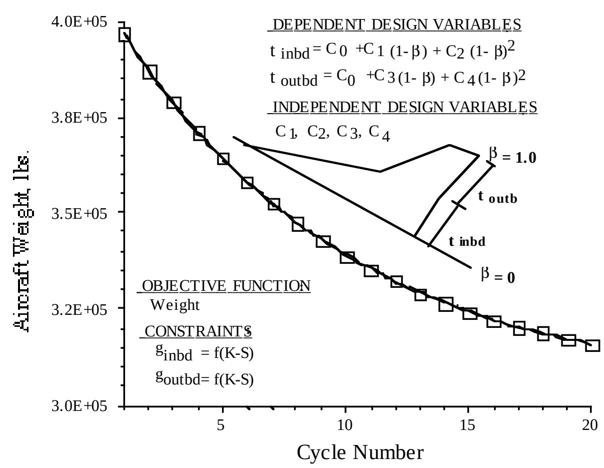

Fig. 18. Weight history for HSCT design optimization.

The spanwise distribution of the skin thickness for the baseline configuration (top curve) and the reduction in the thickness distributions are shown in Fig. 19 after five cycles of the design process.

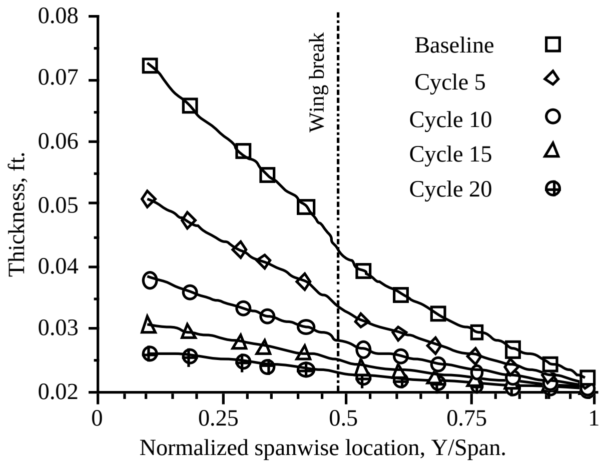

Fig. 19. Thickness history for HSCT design optimization.

The FIDO system has been designed to be adaptable to any distributed computing problem. The above problem demonstrates how a distributed computing system can be utilized in an MDO problem.

CONCLUDING REMARKS

An overview of multidisciplinary optimization (MDO) conceptual elements was presented with two examples of MDO methodology applied to preliminary design problems. The two examples demonstrate areas in which the MDO methodology can make an impact in preliminary design. The continual improvement in computers, communication networks, and the worldwide web will improve the tools available for multidisciplinary computations. However, the computational times required for most preliminary analysis methods still prohibit the use of MDO in a practical design environment. More advanced approximation methods are needed that can temporarily replace expensive analyses and automatically decide when an approximation should be updated to maintain its accuracy.

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